MENIIT
NEET | IIT-JEE | FOUNDATION

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JEE Advanced : Paper-2 (2012)

IMPORTANT INSTRUCTIONS

A. General:

1. This booklet is your Question paper. Do not break the seats of his booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator.
6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains 28 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS:

9. The ORS also has a **CODES** printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET.** Sign at the place provided on the ORS affirming that you have verified that all the code are same.
11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS(R4).

C. Question paper format and Marking scheme:

The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. **Section I** contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
13. **Section II** contains **5 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE are correct**.
14. **Section III** contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

D. Marking Scheme

15. For each question in **Section I**, you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero marks** if no bubbles are darkened. In all other cases, **minus one (–1)** mark will be awarded in this section.
16. For each question in **Section II**, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.
17. For each question in **Section III**, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

PART A : PHYSICS

SECTION-I

(Single Correct Answer Type)

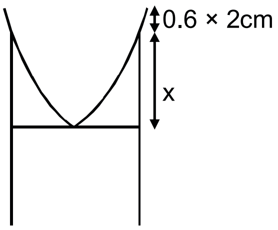
This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(A) 14.0 cm (B*) 15.2 cm (C) 16.4 cm (D) 17.6 cm

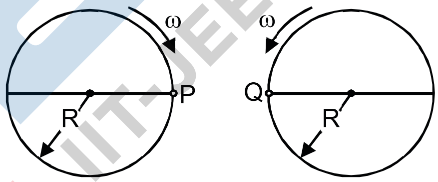
Ans. B

Sol.

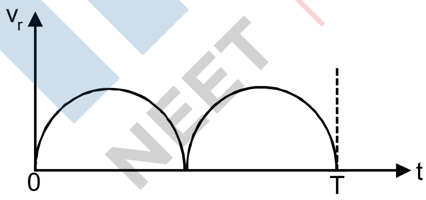


$$1.2 + x = \frac{\lambda}{4} = \frac{5900}{512} = 16.41 \text{]}$$

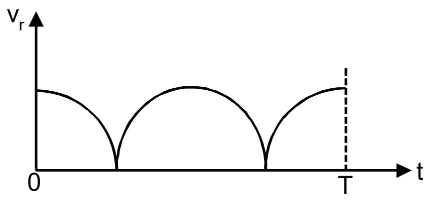
2. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



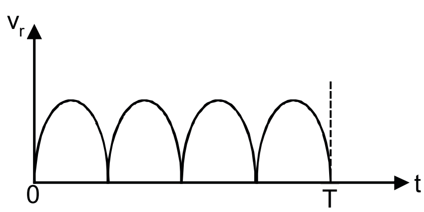
(A*)



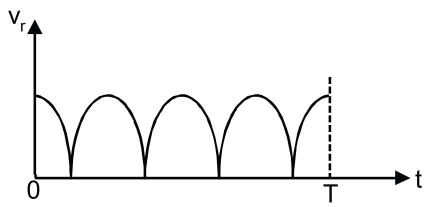
(B)



(C)



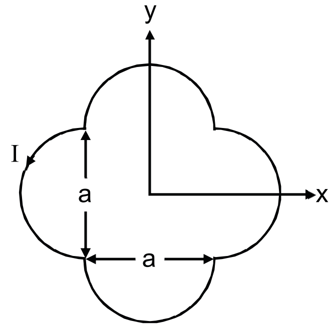
(D)



Ans. A

Sol. At the moment shown, v relative is zero. When they are at diametrically opposite position, relative velocity is zero again and at no other positions.]

3. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

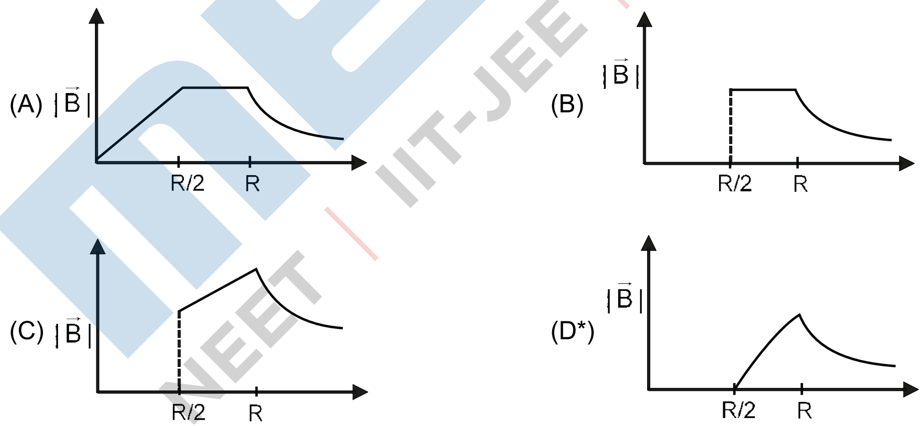


(A) $a^2 I\hat{k}$ (B*) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

Ans. B

Sol. $M = \left(a^2 + \frac{\pi\left(\frac{a}{2}\right)^2}{2}\times 4\right)I = a^2\left(1+\frac{\pi}{2}\right)\times I]$

4. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by



Ans. D

Sol. $B \times 2\pi x = \mu_0 J\left(\pi\left(x^2 - \frac{R^2}{4}\right)\right)$

$B = \frac{\mu_0 J}{2\pi}\left(x - \frac{R^2}{4x}\right)$

5. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

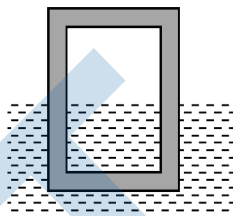
(A*) more than half-filled if ρ_c is less than 0.5

(B) more than half-filled if ρ_c is more than 1.0

(C) half-filled if ρ_c is more than 0.5

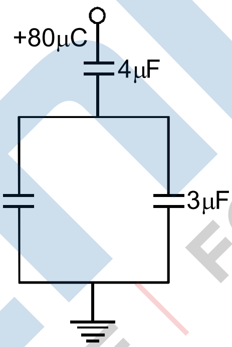
(D) less than half-filled if ρ_c is less than 0.5

Sol. $mg = \rho_w \times \frac{V}{2} g = \rho_w \times V_f g + \rho_C \times V_m \times g$



If the vessel is half filled, the density of the material is half that of water. But if density of the material is less than half of the water, the vessel has to be more than half filled.

6. In the given circuit, a charge of +80μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is



(A) +32 μC (B) +40μC (C*) +48μC (D) +80μC

Ans. C

Sol. $Q_3 = \frac{3}{3+2} \times 80 = 48$

7. Two moles of ideal helium gas are in a rubber balloon at 30°. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol. K)

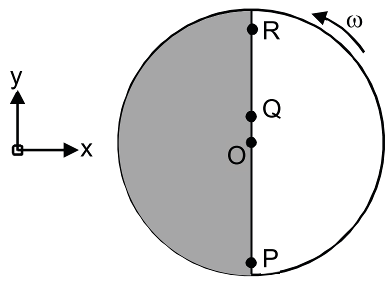
(A) 62 J (B) 104 J (C) 124 J (D*) 208 J

Ans. D

Sol. Isobaric process

$\Delta Q = nC_P \Delta T = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208]$

8. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its center O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region

(B) P lands in the unshaded region and Q in the shaded region

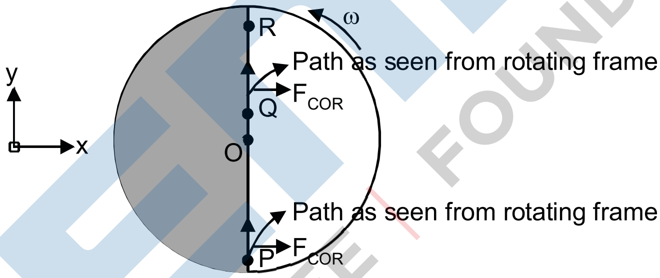
(C*) Both P and Q land in the unshaded region

(D) Both P and Q land in the shaded region

Ans. C

Sol. The velocity of particle at Q is almost entirely in y-z plane. So it will land in unshaded region.

As seen from the rotating frame of reference, particle at P will experience a coriolis force towards the positive x-direction. So it will land in the unshaded region.



$F_{COR} = -2m\,(\vec{\omega} \times \vec{v})$

SECTION-II

(Paragraph Type)

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 9 to 10

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{v}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

9. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(A) $0 \le K \le 0.8 \times 10^6$ eV
(B) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV
(C) $3.0 \text{ eV} \le K < 0.8 \times 10^6$ eV
(D*) $0 \le K < 0.8 \times 10^6$ eV

Ans. D

10. What is the maximum energy of the anti-neutrino?

(A) Zero
(B) Much less than 0.8×10^6 eV
(C*) Nearly 0.8×10^6 eV
(D) Much larger than 0.8×10^6 eV

Ans. C

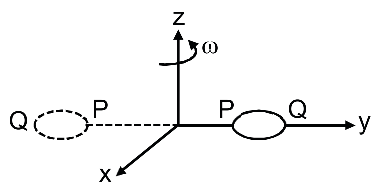
Sol. Q value of reaction slightly decreases because mass of the product increases. So the Q value of the reaction is less than 0.8×10^6 eV.

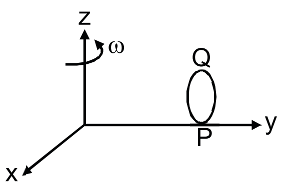
The energy is now shared between the anti-neutrino and the electron. For the maximum of electron, the energy of anti-neutrino is zero and for minimum case, energy of electron is zero.]

Paragraph for Questions 11 and 12

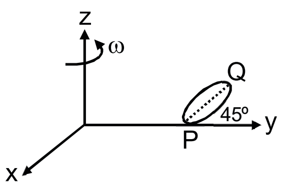
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.

Now consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-z plane ; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.





Case (a)



Case (b)

11. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2}\omega$ for both the cases

(B) It is ω for case (a) ; and $\frac{\omega}{\sqrt{2}}$ for case (b)

(C) It is ω for case (a) ; and $\sqrt{2}\omega$ for case (b)

(D*) It is ω for both the cases

Ans. **D**

12. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(A*) It is vertical for both the cases (a) and (b)

(B) it is vertical for case (a) ; and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(C) It is horizontal for case (a) ; and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

(D) It is vertical for case (a) ; and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

Ans. **A**

Sol. In both the cases in one full revolution, the disc comes back to the original position. So it rotates about its centre of mass with the same angular velocity as ω. Since the orientation of both the disc is the same as the initial position, the axis of rotation is also vertical.]

Paragraph for Questions 13 and 14

Most materials have the refractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Shell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

13. Choose the correct statement

(A) The speed of light in the meta-material is v = c |n|

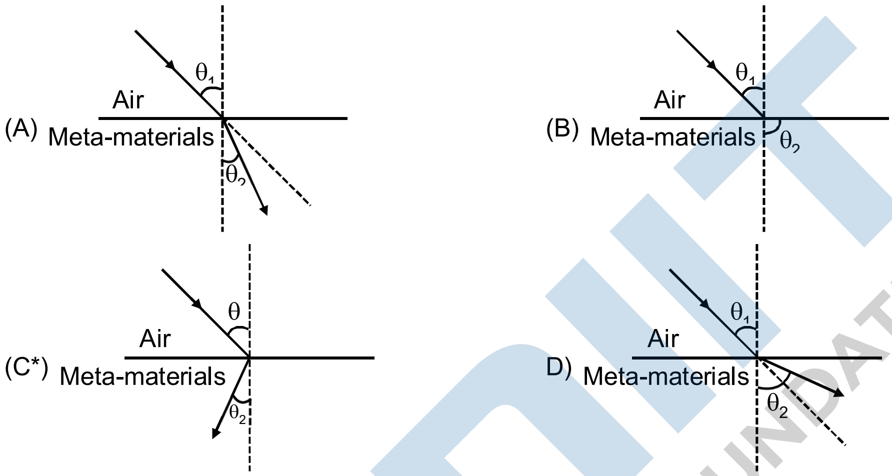
(B*) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is v = c

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}$ |n|, where λ_{air} is the wavelength of the light in air

Ans. B

14. For light incident from air on a meta-material, the appropriate ray diagram is



Ans. C

Sol. Since the physical laws remain the same, the speed of light is lesser in meta-materials.

So $v = \frac{c}{|n|}$

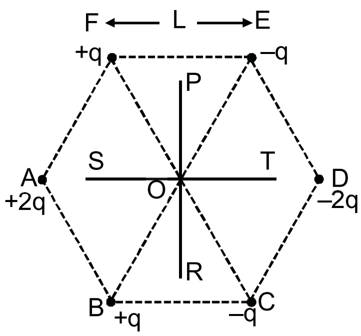
1 sin θ_1 = n sin θ_2. Since n is negative, θ_2 is also negative. So the ray rotates and goes on the opposite side of the normal.]

SECTION-III

(Multiple Correct Answer(s) Type)

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

15. Six-point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is(are) correct?



(A*) The electric field at O is 6K along OD.

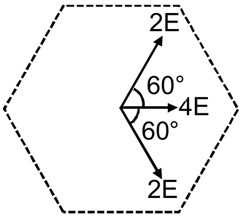
(B*) The potential at O is zero.

(C*) The potential at all points on the line PR is same.

(D) The potential at all points on the line ST is same.

Ans. **[A,B,C]**

Sol. $\therefore E_{res} = 6E = 6\ \frac{1}{4\pi\varepsilon_0}\ \frac{q}{L^2} = 6\ K$



Assuming that potential is zero at ∞

16. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then

(A) $V_Q > V_R > V_P$ (B*) $V_R > V_Q > V_P$ (C) $V_R / V_P = 3$ (D*) $V_P / V_Q = \frac{1}{2}$s

Ans. **B, D**

Sol.

P	Q	R
ρ	ρ	ρ
M	8M	9M
A	4A	
R	2R	$3^{2/3}$ R

$V_P = \sqrt{\frac{GM}{R}}$

$V_Q = \sqrt{\frac{G8M}{2R}} = 2\sqrt{\frac{GM}{R}} = 8^{1/3}\sqrt{\frac{GM}{R}}$

$\therefore \frac{V_P}{V_Q} = \frac{1}{2}$

$V_R = \sqrt{\frac{G(9M)}{3^{2/3}R}} = \sqrt{3^{4/3}\frac{GM}{R}} = 3^{2/3}\sqrt{\frac{GM}{R}} = 9^{1/3}\sqrt{\frac{GM}{R}}$

$\therefore V_R > V_Q > V_P$]

17. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface.

(A*) The point O has a linear velocity $3R\omega\hat{i}$

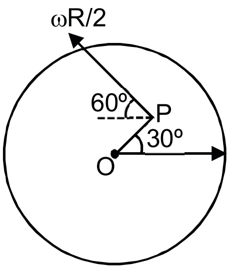
(B*) The point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) The point P has a linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) The point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

Ans. A, B

Sol.



$\vec{v}_0 = 3\omega R(\hat{i})$

$\vec{v}_P = (3\omega R - \frac{\omega R}{2}\cos 60°)\hat{i} + \frac{\omega R}{2}\sin 60°\hat{j}$

18. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time

(B) Cylinder P has larger linear acceleration than cylinder Q

(C) Both cylinders reach the ground with same translational kinetic energy

(D*) Cylinder Q reaches the ground with larger angular speed

Ans. D

Sol. $a = \frac{g\sin\theta}{1+\frac{I_c}{mR^2}}$

since, $I_P > I_Q$

$\therefore$ A & B is not correct & D is correct.

Total KE of both cylinders will be same but translation KE will not be same.]

19. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)

(A*) The emf induced in the loop is zero if the current is constant

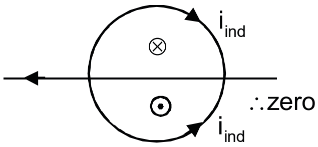
(B) The emf induced in the loop is finite if the current is constant

(C*) The emf induced in the loop is zero if the current decreases at a steady rate

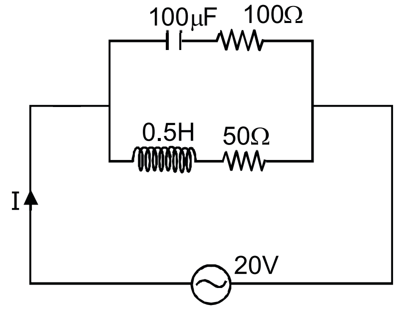
(D) The emf induced in the loop is finite if the current decreases at a steady rate

Ans. A,C

Sol.



22. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)



(A) The current through the circuit, I is 0.3 A

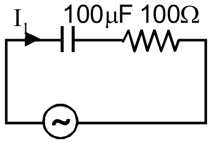
(B) The current through the circuit, I is $0.3\sqrt{2}$A

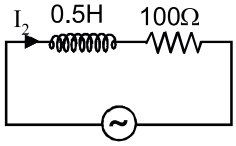
(C) The voltage across 100Ω resistor $= 10\sqrt{2}$V

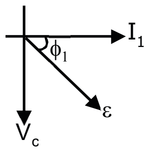
(D) The voltage across 50Ω resistor = 10 V

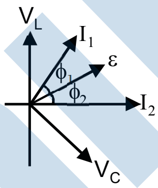
Ans. **[C or A,C]**

Sol.









$\tan\phi_1 = \dfrac{X_C}{R_1} = 1$

$\therefore \phi_1 = 45°$

$I_1 = \dfrac{20}{\sqrt{X_c^2 + R_1^2}} = \dfrac{1}{5\sqrt{2}}$

$\tan\phi_2 = \dfrac{X_L}{R_2} = 1$

$\therefore \phi_2 = 45°$

$I_2 = \dfrac{20}{\sqrt{X_L^2 + R_2^2}} = \dfrac{2}{5\sqrt{2}}$

$\therefore I = \sqrt{I_1^2 + I_2^2} = \dfrac{1}{\sqrt{10}}$

$V_L = I_2 X_L = 10\sqrt{2}$, $V_c = I_1 X_c = 10\sqrt{2}$

PART B : CHEMISTRY

PAPER-2

SECTION -I

(Single Correct Answer Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

21. Using the data provided, calculate the multiple bond energy (kJ mol^{-1}) of a $C \equiv C$ bond in C_2H_2. That energy is (take the bond energy of a C–H bond as 350 kJ mol^{-1}

$2C(s) + H_2(g) \longrightarrow C_2H_2(g)$ $\Delta H = 225$ kJ mol^{-1}

$2C(s) \longrightarrow 2C(g)$ $\Delta H = 1410$ kJ mol^{-1}

$H_2(g) \longrightarrow 2H(g)$ $\Delta H = 330$ kJ mol^{-1}

(A) 1165 (B) 837 (C) 865 (D*) 815

Sol. $\Delta H = \sum(B.E)_{reactants} - \sum(B.E)_{products}$ s

or, $225 = (1410 + 330) - (B.E._{C \equiv C} + 2 \times 350)$

$\therefore$ B.E. $_{C\equiv C}$ = 815 kJ / mol

22. The major product H of the given reaction sequence is :

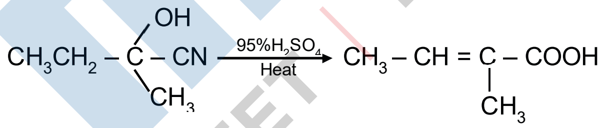
$CH_3-CH_2-CO-CH_3 \xrightarrow{^{\ominus}CN} G \xrightarrow[Heat]{95\% H_2SO_4} H$

(A) $CH_3-CH=C(CH_3)-COOH$

(B) $CH_3-CH=C(CH_3)-CN$

(C) $CH_3-CH=C(OH)(CH_3)-COOH$

(D) $CH_3-CH=C(CH_3)-CO-NH_2$

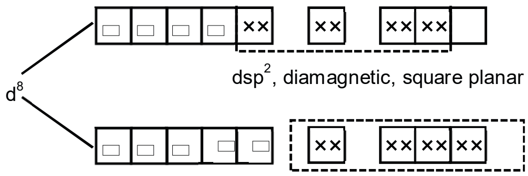
Sol. 

Reference Page No. 744, 7th edition Solomon

23. $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behavior (paramagnetic/diamagnetic). The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

(A) tetrahedral and tetrahedral
(B) square planar and square planar
(C*) tetrahedral and square planar
(D) square planar and tetrahedral

Sol. $Ni^{+2} \longrightarrow d^8$ configuration



sp^2, paramagnetic, tetrahedral

24. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agents used are:

(A) O_2 and CO respectively (B*) O_2 and Zn dust respectively

(C) HNO_3 and Zn dust respectively (D) HNO_3 and CO respectively

Sol. $Ag_2S + 4NaCN \ \ell \ Na_2S + 2Na[Ag(CN)_2]$

$4Na_2S + 5O_2$ (O.A.) $+ 2H_2O \longrightarrow 2Na_2SO_4 + 4NaOH + 2S$

$2Na[Ag(CN)_2]$ + Zn (R.A.) $\longrightarrow Na_2[Zn(CN)_4] + 2Ag$

25. The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type ; the oxidation states of phosphorus in phosphine and the other product are respectively :

(A) redox reaction ; – 3 and – 5 (B) redox reaction; + 3 and +5

(C*) disproportionation reaction ; – 3 and +5 (D) disproportionation reaction ; –3 and + 3

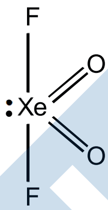
Sol. $3NaOH + P_4 + 3H_2O \longrightarrow PH_3(-3) + 3NaH_2PO_2(+1)$

$H_2PO_2^- + 2OH^- \longrightarrow 2H_2 + PO_4^{3-}$ (+5)

26. The shape of XeO_2F_2 molecule is :

(A) trigonal bipyramidal (B) square planar

(C) tetrahedral (D*) see-saw

Sol. 

sp^3d (4σ + 1 ℓ.p.) See-saw

27. For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2ºC. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is

(take K_b = 0.76 K kg mol^{-1})

(A*) 724 (B) 710 (C) 736 (D) 718

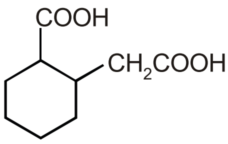
Sol. $\Delta T_b = K_b \cdot m$

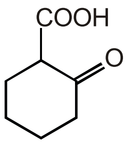
$$\text{or, } 2 = 0.76 \times \frac{2.5 \times 1000}{M \times 100} \Rightarrow M = \frac{0.76 \times 25}{2}$$

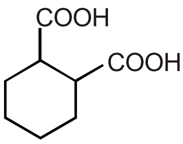
$$\text{Now, } \frac{P^\circ - P}{P^\circ} = \frac{n_{solute}}{n_{solvent}} (n_{solvent} >> n_{solute})$$

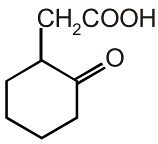
$$\text{or, } \frac{760 - P}{760} = \frac{\left(\frac{2.5}{M}\right)}{\left(\frac{100}{18}\right)} \Rightarrow P = 724 \text{ mm Hg}$$

28. The compound that undergoes decarboxylation most readily under mild condition is :

(A) 

(B*) 

(C) 

(D) 

Sol. β-Keto acids undergoes decarboxylation most readly.

SECTION - II

(Paragraph Type)

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choice (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 29 & 30

The electrochemical cell shown below is a concentration cell.

$M|M^{2+}$ (saturated solution of a sparingly soluble salt, MX_2) || M^{2+} (0.001 mol dm^{-3}) | M

The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes.

The emf of the cell at 298 K is 0.059 V.

29. The value of ΔG (kJ mol^{-1}) for the given cell is (take 1 F = 96500 C mol^{-1})

(A) – 5.7 (B) 5.7 (C) 11.4 (D*) –11.4

Sol. The cell reaction is

Left electrode: M (s) ℓ M^{2+} (s M) + 2 e^-

Right electrodes : M^{2+} (0.001 M) + 2e^- ℓ M (s)

Net cell reaction : M(s) + M^{2+} (0.001 M) ℓ M^{2+}(s M) + M (s)

$\Delta G = - nFE_{cell} = - 2 \times 96500 \times 0.059$

$= - 11387 \text{ J mol}^{-1} \approx - 11.4 \text{ kJ mol}^{-1}$

30. The solubility product (K_{sp} ; mol^3 dm^{-9}) of MX_2 at 298 K based on the information available for the given concentration cell is (take 2.303 × R × 298/F = 0.059 V)

(A) 1 × 10^{-15} (B*) 4 × 10^{-15} (C) 1 × 10^{-12} (D) 4 × 10^{-12}

Sol. $E_{cell} = E^\circ_{cell} - \frac{2.303RT}{nF} \cdot \log Q$

or $0.0509 = 0 - \frac{0.059}{2}.\log\frac{S}{0.001}$

$\therefore$ S = 10^{-5} M

Now, MX_2 (s) ℓ M^{2+} (aq) + 2 X^- (aq)

S M 2 SM

$\therefore K_{sp} = [M^{2+}][X^-]^2 = S.(2s)^2 = 4s^3$

$= 4 \times 10^{-15}$ M^3

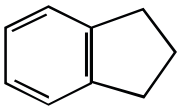
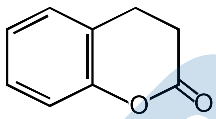
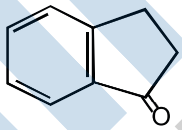
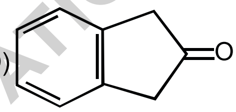
Paragraph for Questions 31 & 32

In the following reaction sequence, the compound J is an intermediate.

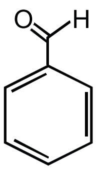
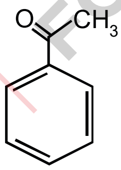
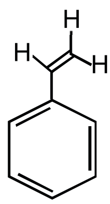
$$I \xrightarrow[CH_3COONa]{(CH_3CO)_2O} J \xrightarrow[\text{(ii) } SOCl_2 \quad \text{(iii) anhyd. } AlCl_3]{\text{(i) } H_2, Pd/C} K$$

J ($C_9H_6O_2$) gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test.

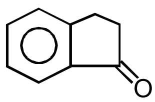
31. The compound 'K' is :

(A)  (B)  (C*)  (D) 

32. The compound 'I' is :

(A*)  (B)  (C)  (D) 

Sol.

Ph–CH=0 $\xrightarrow[\text{Perkinreaction}]{(CH_3CO_2)O,\ CH_3COONa}$ Ph–CH=CH–COOH $\xrightarrow{H_2Pd/C}$ $PhCH_2CH_2$–COOH $\xrightarrow{SOCl_2}$ $PhCH_2CH_2COCl$ $\xrightarrow[\text{Friedel Craft reaction}]{\text{anhydAlCl}_3}$ 

Paragraph for Questions 33 to 34

Bleaching powder and bleach solution are produced on a large scale and used in several household products. The effectiveness of bleach solution is often measured by iodometry.

33. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is :

(A) 0.48 M (B) 0.96 M (C*) 0.24 M (D) 0.024 M

Sol. $OCl^- + I^- \longrightarrow Cl^- + I_2$

(From bleaching powder)

$I_2 + S_2O_3^{2-} \longrightarrow I^- + S_4O_6^{2-}$

n_{eq} bleaching = n_{eq} $S_2O_3^{2-}$

or, $\frac{25 \times (S \times 2)}{1000} = \frac{48 \times 0.25}{1000}$

$\therefore$ S = 0.24 M

34. Bleaching powder contains a salt of an oxoacid as one its components. The anhydride of that oxoacid is:

(A*) Cl_2O (B) Cl_2O_7 (C) ClO_2 (D) Cl_2O_6

Sol. Bleaching powder is Ca(OCl)Cl

Cl_2O is anhydride of HOCl

$Cl_2O + H_2O \longrightarrow 2HOCl$

SECTION -III

(Multiple Correct Answer(s) Type)

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE may be correct.

35. For the given aqueous reactions, which of the statement(s) is (are) true?

excess KI + $K_3[Fe(CN)_6]$ $\xrightarrow{\text{dilute } H_2SO_4}$ brownish-yellow solution

$\downarrow ZnSO_4$

white precipitate + brownish – yellow filtrate

(filtrate) $\downarrow Za_2S_2O_3$

colourless solution

(A*) The first reaction is a redox reaction

(B) White precipitate is $Zn_3[Fe(CN)_6]_2$

(C*) Addition of filtrate to starch solution gives blue colour

(D*) White precipitate is soluble in NaOH solution.

Sol. $3KI + K_3[Fe(CN)_6] \longrightarrow KI_3 + K_4[Fe(CN)_6]$

(brownish yellow)

$3Zn^{2+} + 2K^+ + 2[Fe(CN)_6]^{4-} \longrightarrow K_2Zn_3[Fe(CN)_6]_2$ (white ppt)

KI_3 will remain unreacted

$KI_3 + 2Na_2S_2O_3 \longrightarrow 2NaI + KI + Na_2S_4O_6$

$K_2Zn_3[Fe(CN)_6]_2 + 12OH^- \longrightarrow 2[Fe(CN)_6]^{4-} + [Zn(OH)_4]^{2-} + 2K^+$

Starch + $KI_3 \rightarrow$ Starch – Iodine complex (blue colour)

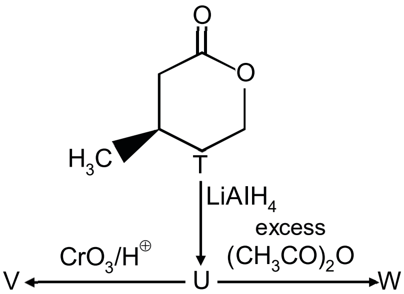
36. With reference to the scheme given, which of the given statement(s) about T, U, V and W is (are) correct?

(A*) T is soluble in hot aqueous NaOH

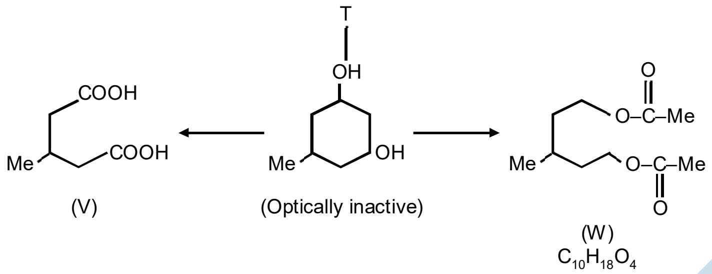
(B) U is optically active

(C*) Molecular formula of W is $C_{10}H_{18}O_4$

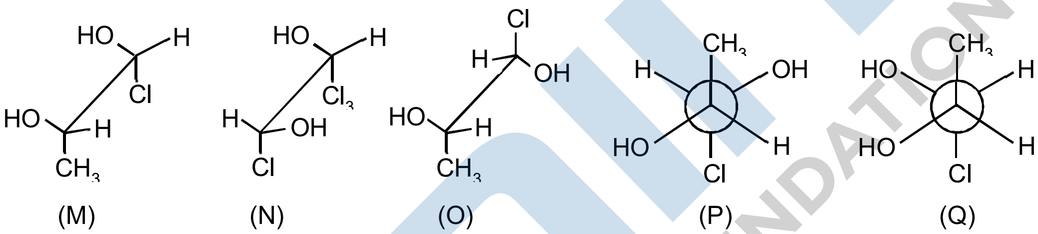
(D*) V gives effervescence on treatment with aqueous $NaHCO_3$



Sol.



37. Which of the given statement(s) about N, O, P and Q with respect to M is (are) correct?



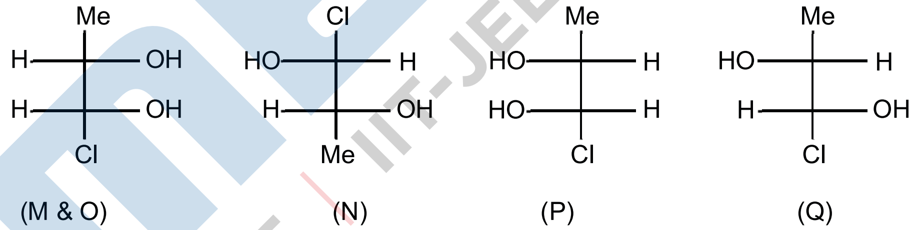
(A*) M and N are non-mirror image stereoisomers

(B*) M and O are identical

(C*) M and P are enantiomers

(D) M and Q are identical

Sol.



M and Q are diastereoisomers.

38. With respect to graphite and diamond, which of the statement(s) given below is (are) correct?

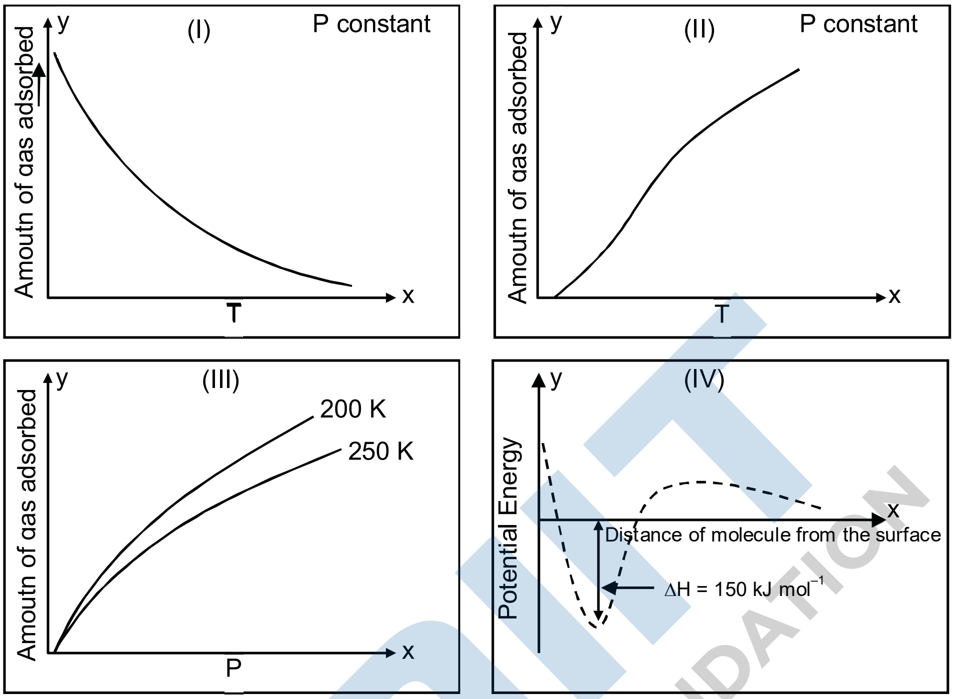
(A) Graphite is harder than diamond

(B*) Graphite has higher electrical conductivity than diamond.

(C) Graphite has higher thermal conductivity than diamond.

(D*) Graphite has higher C–C bond order than diamond.

39. The given graphs / data I, II, III and IV represents general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about I, II, III and IV is (are) correct?



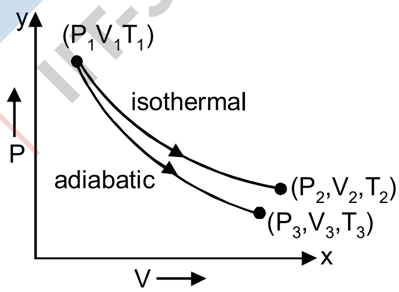
(A*) I is physisorption and II is chemisorption

(B) I is physisorption and III is chemisorption

(C*) IV is chemisorption and II is chemisorption

(D) IV is chemisorption and III is chemisorption

40. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statements(s) is (are) correct?



(A*) $T_1 = T_2$ (B) $T_3 > T_1$ (C) $w_{isothermal} > w_{adiabatic}$ (D*) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

Ans. If magnitude is completed, then C is also correct.

AD or ACD

PART C : MATHEMATICS

SECTION-I

(Single Correct Answer Type) [3 Marks]

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

41. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is

(A) 0 (B*) $\frac{\pi^2}{2}-4$ (C) $\frac{\pi^2}{2}+4$ (D) $\frac{\pi^2}{2}$

Sol. Let I = $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$

$x-\sqrt{2}\,y = 1-\sqrt{2}$

$I = 2\int_0^{\pi/2}\underbrace{x^2}_{I}\,\underbrace{\cos x}_{II}\,dx = 2\left[x^2(\sin x)-2x(-\cos x)+2(-\sin x)\right]_0^{\pi/2}$

Applying integration by parts

I = $\frac{\pi^2}{2}-4$. **Ans.**]

42. Let a_1, a_2, a_3 .... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D*) 25

Sol. $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$ .... $\frac{1}{a_{20}}$ Arithmetic Progression

$\Rightarrow \frac{1}{5}, \frac{1}{a_2}, \frac{1}{a_3}$ .... $\frac{1}{25}$ Arithmetic Progression

$\therefore \frac{1}{25}=\frac{1}{5}+19d \Rightarrow d=\frac{-4}{25.19}$ Let n^{th} term of Arithmetic progression $\frac{1}{a_n}$ is negative

$\therefore \frac{1}{a_n}=\frac{1}{5}-\frac{4(n-1)}{25\cdot 19}<0 \Rightarrow (n-1)\,4 > 19\cdot 5$

$\Rightarrow n > \frac{99}{4}$ (less than 25) $\Rightarrow$ n = 25. **Ans.**]

43. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, – 1) is

(A*) 5x – 11y + z = 17 (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}\,y = 1 - \sqrt{2}$

Sol. Let the plane passes through the line of intersection of given place is

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow x(1 + \lambda) + y(2 - \lambda) + z(3 + \lambda) - (2 + 3\lambda) = 0$ ....(1)

Its distance from (3, 1, –1) is $\frac{2}{\sqrt{3}}$.

$$\Rightarrow \frac{3(1+\lambda)+(2-\lambda)-(3+\lambda)-(2+3\lambda)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}} = \pm\frac{2}{\sqrt{3}}$$

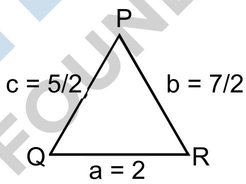
$$\Rightarrow 3\lambda^2 = (1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2 \Rightarrow \lambda = \frac{-7}{2}$$

Put value of λ in equation (1) we get $5x - 11y + z = 17$. **Ans.**]

44. Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C*) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Sol. $\cos P = \frac{\frac{25}{4}+\frac{49}{4}-4}{2\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)} = \frac{74-6}{70} = \frac{58}{70} = \frac{29}{35}$



Now, $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P} = \frac{1-\cos P}{1+\cos P}$

$$= \frac{1-\frac{29}{35}}{1+\frac{29}{35}} = \frac{6}{64} = \frac{3}{32} = \left(\frac{3}{4\Delta}\right)^2 \Rightarrow \text{(C) is correct.}$$

As, $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{4(2)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)} = \sqrt{6}$. **Ans.**]

45. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0 (B) 3 (C*) 4 (D) 8

Sol. Let $(2\hat{i}+3\hat{j}+4\hat{k})$ be $\vec{r}$.

Hence, = $\vec{a}\times\vec{r} = \vec{r}\times\vec{b} \Rightarrow (\vec{a}+\vec{b})\times\vec{r} = 0 \Rightarrow (\vec{a}+\vec{b}) = \lambda\vec{r}$

$\lambda = \pm 1 \quad \because |\vec{a}+\vec{b}| = \sqrt{29}$

Hence, $(\vec{a}+\vec{b}) = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$\Rightarrow (\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm 4$. **Ans.**]

46. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D*) PX = – X

Sol. $P^T = 2P + I$

$\Rightarrow (P^T)^T = 2P^T + I^T$ or $P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$ or $P = -I \Rightarrow PX = -X$. **Ans.**]

47. Let α (a) and β (a) be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right) = 0$, where a > – 1. Then $\lim_{a\to 0^+} \alpha(a)$ and $\lim_{a\to 0^+} \beta(a)$ are

(A) $\frac{-5}{2}$ and 1 (B*) $\frac{-1}{2}$ and –1 (C) $\frac{-7}{2}$ and 2 (D) $\frac{-9}{2}$ and 3

Sol. $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right) = 0$

Let $1 + a = t^6$ ....(i)

when $a \to 0^+ \Rightarrow t \to 1$

∴ Given equation becomes

$(t^2 - 1)x^2 + (t^3 - 1)x + (t - 1) = 0$

$(t + 1)x^2 + (t^2 + t + 1)x + 1 = 0 \Rightarrow 2x^2 + 3x + 1 = 0 \Rightarrow x = -1$ or $\frac{-1}{2}$

∴ $\lim_{a\to 0^+} \alpha(0) = -1$ and $\lim_{a\to 0^+} \beta(0) = \frac{-1}{2}$. **Ans.**]

48. Four fair dice D_1, D_2, D_3, and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A*) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Sol. **Case I :** D_4 and D_1 (or D_2 or D_3) show same digit, then number of ways = 6 × 5 × 5 × 3

Case II : D_4 and D_1, D_2 (or D_2D_3 or D_3D_1) show same digit , then number of ways = 6 × 5 × 3

Case III : D_1, D_2, D_3, D_4 all show same digit, then number of ways = 6

$\Rightarrow P = \frac{450 + 90 + 6}{6^4} = \frac{91}{216}$. **Ans.**

Aliter: P (D_3 show one of D_1, D_2, D_3 number) = P(All three D_1, D_2, D_3 shows same number)

+ P (Any two of D_1, D_2, D_3 shows same number and third shows different)

+ P (All three D_1, D_2, D_3 shows different number)

$$= {}^6C_1\left(\frac{1}{6}\right)^3\left(\frac{1}{6}\right) + {}^6C_2 \cdot {}^3C_2 \cdot 2!\left(\frac{1}{6}\right)^3\left(\frac{2}{6}\right) + {}^6C_3 \cdot 3!\left(\frac{1}{6}\right)^3\left(\frac{3}{6}\right)$$

$$= \frac{1}{6^4}[6 + 15 \times 12 + 6 \times 60] = \frac{91}{216}.$$ **Ans.]**

SECTION-II

(Paragraph Type) [3 Marks]

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question no. 49 and 50

Let $f(x) = (1 - x)^2 \sin^2 x + x^2 \; \forall \; x \in R$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ell n t\right) f(t)\, dt \;\; \forall \; x \in (1, \infty)$.

49. Consider the statements :

P : There exists some $x \in R$ such that $f(x) + 2x = 2(1 + x^2)$.

Q : There exist some $x \in R$ such that $2f(x) + 1 = 2x(1 + x)$.

Then

(A) both **P** and **Q** are true

(B) **P** is true and **Q** is false

(C*) **P** is false and **Q** is true

(D) both **P** and **Q** are false

50. Which of the following is true?

(A) g is increasing on $(1, \infty)$.

(B*) g is decreasing on $(1, \infty)$.

(C) g is increasing on (1, 2) and decreasing on $(2, \infty)$.

(D) g is decreasing on (1, 2) and increasing on $(2, \infty)$.

Sol.

(i) Let $F(x) = f(x) + 2x - 2 - 2x^2$

$F(x) = (1 - x)^2 \sin^2 x + x^2 + 2x - 2 - 2x^2$

$F(x) = (1 - x)^2 \sin^2 x - (1 - x)^2 - 1$

$F(x) = -(1 - x)^2 \cos^2 x - 1 = -1\,[(1 - x)^2 \cos^2 x + 1]$

$F(x) < 0 \; \forall \; x \in R \Rightarrow F(x) = 0$ is not possible for any $x \in R$. So **P** is false

Let $G(x) = 2f(x) + 1 - 2x - 2x^2$

$G(x) = 2(1 - x)^2 \sin^2 x + 2x^2 + 1 - 2x - 2x^2$

$G(x) = 2\,[1 - 2x + x^2] \sin^2 x + (1 - 2x)$

$G(x) = (2\sin^2 x + 1)(1 - 2x) + 2x^2 \sin^2 x$

Now G (x) = 0 $\Rightarrow \sin^2 x = \dfrac{2x-1}{2(x-1)^2}$

As $\dfrac{2x-1}{2(x-1)^2} \in [-1/2, \infty)$, so **Q** is true.

Hence **P** is false and **Q** is true.

(ii) $g(x) = \int_1^x \left(\dfrac{2(t-1)}{t+1} - \ln t\right) f(t)\,dt$

$g'(x) = \left(\dfrac{2(x-1)}{x+1} - \ln x\right) f(x)$

$g'(x) = \left[2 - \dfrac{4}{(x+1)} - \ln x\right] f(x)$

Consider $h(x) = 2 - \dfrac{4}{x+1} - \ln x$

$h'(x) = \dfrac{4}{(x+1)^2} - \dfrac{1}{x}$

$h'(x) = \dfrac{4x - x^2 - 2x - 1}{x\,(x+1)^2}$

$h'(x) = \dfrac{-(x-1)^2}{x\,(x+1)^2} < 0$

$\therefore$ h(x) is a decreasing function $\forall\ x \in (1, \infty)$

$h(x) < h(1) = 0$

$\therefore$ $h(x) < 0\ \forall\ x \in (1, \infty)$

$\therefore$ $g(x) < 0$ as $f(x) > 0\ \forall\ x \in R \Rightarrow g(x)$ is decreasing in $x \in (1, \infty)$. **Ans.**]

Paragraph for Question no. 51 and 52

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point P $\left(\sqrt{3}, 1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$.

51. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}\,y = 4$ (D*) $x + 2\sqrt{2}\,y = 6$

52. A possible equation of L is

(A*) $x - \sqrt{3}\,y = 1$ (B) $x + \sqrt{3}\,y = 1$ (C) $x - \sqrt{3}\,y = -1$ (D) $x + \sqrt{3}\,y = 5$

Sol.

(ii) Equation of tangent at P($\sqrt{3}$, 1)

on $x^2 + y^2 = 4$ is $\sqrt{3}x + y = 4$ ....(1)

line perpendicular to equation (1)

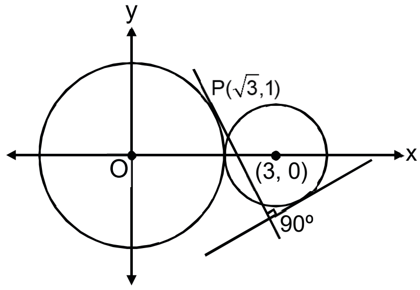
$x - \sqrt{3}y + \lambda = 0$ ....(2)

Perpendicular from (3, 0) on (2) = unity

$\left|\frac{3+\lambda}{2}\right| = 1 \Rightarrow \lambda + 3 = \pm 2$

$\Rightarrow \lambda = -1$ or $\lambda = 5$

$\therefore$ Equation of L is $x - \sqrt{3}y - 1 = 0 \Rightarrow$ A



(i) $\frac{C}{C-3} = \frac{2}{1}$

$2C - 6 = C$

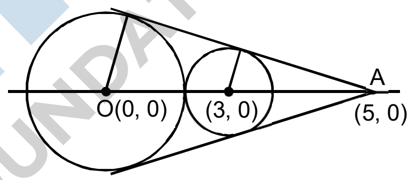
$\Rightarrow C = 6$

Hence, A (6, 0)

Equation of a line through A is $y = m(x - 6)$ ....(3)

$\Rightarrow mx - y - 6m = 0$

equation (3) is tangent to both, hence $\left|\frac{3m - 6m}{\sqrt{1+m^2}}\right| = 1$



$\Rightarrow 9m^2 = 1 + m^2 \Rightarrow m = \frac{1}{2\sqrt{2}}$ or $\frac{-1}{2\sqrt{2}}$

$\therefore$ Common tangent is $y = \frac{1}{2\sqrt{2}}(x - 6) \Rightarrow x - \sqrt{2}\,2y = 6$

or $-2\sqrt{2}y = x - 6 \Rightarrow x + 2\sqrt{2} = 6$. **Ans.**]

Paragraph for Question no. 53 and 54

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

53. Which of the following is correct?

(A*) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

54. The value of b_6 is

(A) 7 (B*) 8 (C) 9 (D) 11

Sol.

(i) a_n = numbers of all n digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are zero.

$\Rightarrow$ the number will end with 0 or 1

Case-I: If the number ends with 0, then

$(n-1)^{th}$ digit should be 1

Hence number of such numbers will be b_{n-1}.

b_{n-1}	1	0
	$(n-1)^{th}$	n^{th}

Case-II: If the number ends with 1, then

First (n – 1) digits should be (n – 1) digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are zero.

Hence number of such numbers will be a_{n-1}.

a_{n-1}	1
	n^{th}

$\therefore a_n = a_{n-1} + b_{n-1}$ ....(1)

but b_n is the number of such numbers which will end with 1

$\therefore$ first (n – 1) digits will be (n – 1) digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are zero.

Hence $b_n = a_{n-1}$

From (1), we get

$a_n = a_{n-1} + a_{n-2}$ and for n = 17, $a_{17} = a_{16} + a_{15}$ $\Rightarrow$ **(A)**

c_n is the number of such numbers ending with 0.

$\therefore$ $(n-1)^{th}$ digit should be 1.

first (n – 1) digits will be (n – 1) digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are zero which will end with 1.

$\therefore$ $c_n = b_{n-1}$.

(ii) $b_6 = a_5 = a_4 + a_3 = 2a_3 + a_2 = 3a_2 + 2a_1$

$a_2 = 2$ (10 and 11)

$a_1 = 1$ (1)

$\therefore b_6 = 8$]

SECTION-III

(Multiple Correct Answer(S) Type) [4 Marks]

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE is correct.

55. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$ (B*) $y + z = -1$ (C*) $y - z = -1$ (D) $y - 2z = -1$

Sol. Plane containing the given lines is $\begin{vmatrix} x-1 & y+1 & z \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$

$\Rightarrow (x - 1)(k^2 - 4) - (y + 1)(2k - 10) + z(4 - 5k) = 0$ ....(1)

which also contains the point (–1, – 1, 0)

$-2(k^2 - 4) - 0 + 0 = 0 \Rightarrow k = \pm 2$

Putting k = 2 in (1)

$0 - (y + 1)(-6) + z(-6) = 0 \Rightarrow y + 1 - z = 0$

Again putting k = – 2 in (1)

$0 - (y + 1)(-14) + z(14) = 0 \Rightarrow y + 1 + z = 0$. **Ans.**]

56. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is(are)

(A*) – 2 (B) – 1 (C) 1 (D*) 2

Sol. adj(P) = $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{adj}(P)| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 4$

$|\text{adj}(P)| = |P|^2 = 4 \Rightarrow P = \pm 2$. **Ans.**]

57. Let $f : (-1, 1) \to R$ be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is(are)

(A) $1 - \sqrt{\frac{3}{2}}$ (B*) $1 + \sqrt{\frac{3}{2}}$ (C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$

Ans. **zero marks to all**

Sol. $\cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

But $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

$f(\cos 4\theta) = \frac{2\cos^2\theta}{2\cos^2\theta - 1} \Rightarrow \frac{1+\cos 2\theta}{\cos 2\theta} = 1 + \sec 2\theta$

$f(\cos 4\theta) = 1 + \sqrt{\frac{3}{2}}$ or $1 - \sqrt{\frac{3}{2}}$. **Ans.**]

58. Let X and Y be two events such that $P(X / Y) = \frac{1}{2}$, $P(Y / X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is(are) correct?

(A*) $P(X \cup Y) = \frac{2}{3}$

(B*) X and Y are independent

(C) X and Y are not independent

(D) $P(X^c \cap Y) = \frac{1}{3}$

Sol. $P(X / Y) = \frac{1}{2}$

$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cap Y) = P(X) \cdot P(Y) \Rightarrow$ X, Y are independent

$P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

$P(X^c \cup Y) = P(Y) - P(X \cap Y) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$. **Ans.**]

59. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

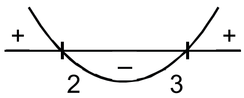
(A*) f has a local maximum at x = 2

(B*) f is decreasing on (2, 3)

(C*) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D*) f has a local minimum at x = 3.

Sol. $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$



Sign Scheme of f'(x)

$f'(x) = e^{x^2}(x-2)(x-3) = 0 \Rightarrow x = 2, 3$ are the critical point

$f'(x) = 0$ at 2 and $3 \Rightarrow f''(x)$ will have atleast one root in (2, 3) by **Rolle**'s Theorem.]

60. For every integer n, let a_n and b_n be real numbers. Let function f : R → R be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers n.}$$

If f is continuous, then which of the following hold(s) for all n?

(A) $a_{n-1} - b_{n-1} = 0$

(B*) $a_n - b_n = 1$

(C) $a_n - b_{n+1} = 1$

(D*) $a_{n-1} - b_n = -1$

Sol. Since, f(x) is continuous for all x at x = 2n, $n \in I$

LHL = $\lim_{h \to 0} (b_n + \cos \pi(2n-h)) = b_n + 1$

RHL = $\lim_{h \to 0} (a_n + \sin \pi(2n+h)) = a_n$

Also, $f(2n) = a_n$.

$\therefore a_n = b_n + 1 \Rightarrow a_n - b_n = 1$. **Ans.**]